Chapter 4

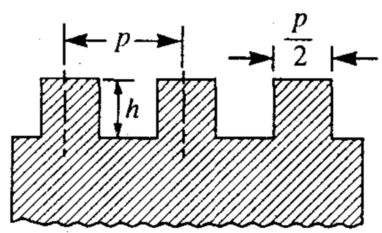
Power Screws

4.1 Introduction

- The power screws (also known as translation screws) are used to convert rotary motion into translatory motion.
- For example, in the case of the lead screw of lathe, the rotary motion is available but the tool has to be advanced in the direction of the cut against the cutting resistance of the material.
- In case of screw jack, a small force applied in the horizontal plane is used to raise or lower a large load.
- Power screws are also used in vices, testing machines, presses, etc.
- In most of the power screws, the nut has axial motion against the resisting axial force while the screw rotates in its bearings.
- In some screws, the screw rotates and moves axially against the resisting force while the nut is stationary and in others the nut rotates while the screw moves axially with no rotation.

4.2 Types of Screw Threads used for Power Screws

1. Square thread.

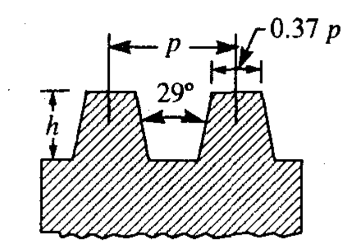


$h = 0.5\,p$

(a) Square thread.

- A square thread, as shown in Fig.(a) is adapted for the transmission of power in either direction.
- This thread results in maximum efficiency and minimum radial or bursting pressure on the nut.
- It is difficult to cut with taps and dies.
- It is usually cut on a lathe with a single point tool and it can not be easily compensated for wear. .
- Strength of square threads is lowest of all the thread forms.
- Engagement and disengagement is difficult
- The square threads are employed in screw jacks, presses and clamping devices.

2. Acme or trapezoidal thread.



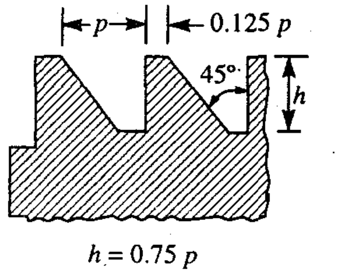
$h = 0.5\,p + 0.25$ mm

(b) Acme thread.

- An acme or trapezoidal thread, as shown in Fig. (b) is a modification of square thread.
- The slight slope given to its sides which lowers the efficiency slightly than square thread

- The slope on threads also introduces some bursting pressure on the nut, but increases its area in shear.
- It is used where a split nut is required and where provision must be made to take up wear as in the lead screw of a lathe.
- Wear may be taken up by means of an adjustable split nut.
- An acme thread may be cut by means of dies and hence it is more easily manufactured than square thread.
- Angle between flanks is 29^0 for acme threads and 30^0 for trapezoidal treads.

<u>3. Buttress thread</u>.

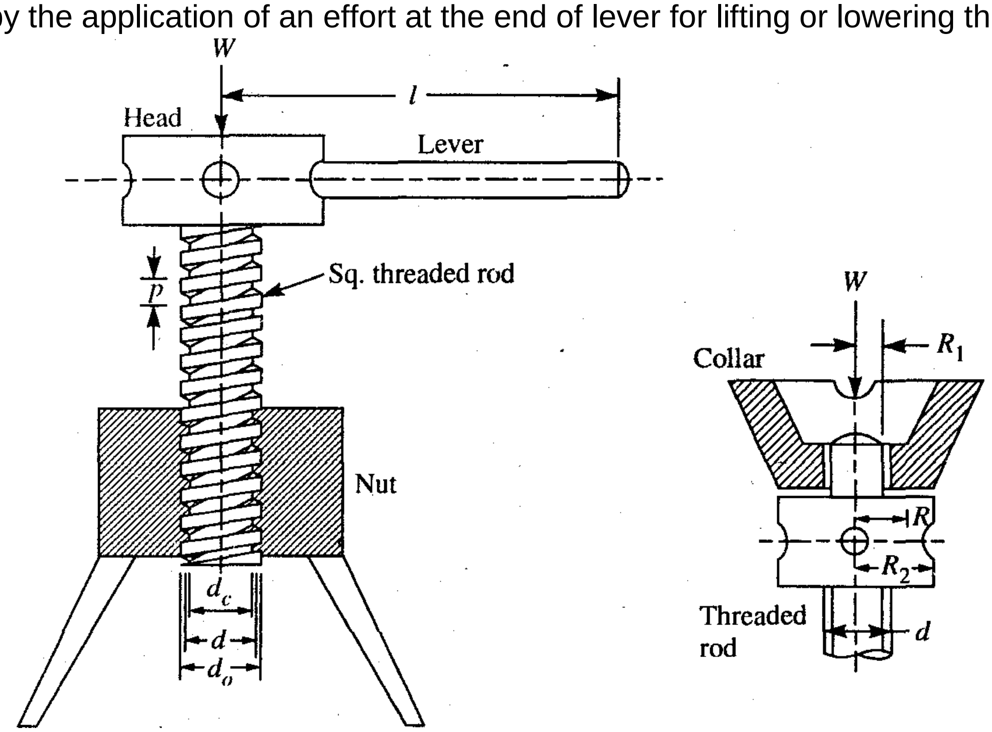


(c) Buttress thread.

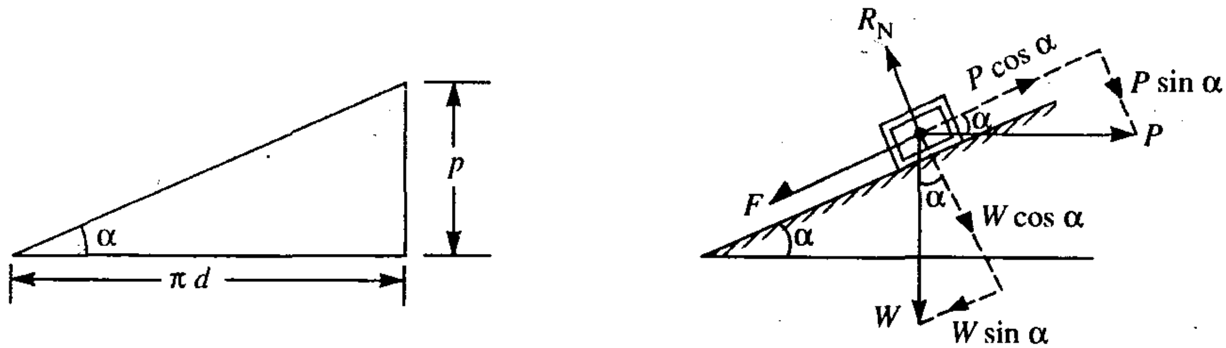
- A buttress thread, as shown in Fig. (c), is used when large forces act along the screw axis in one direction only.
- This thread combines the higher efficiency of square thread and the ease of cutting and the adaptability to a split nut of acme thread.
- It is stronger than other threads because of greater thickness at the base of the thread.
- The buttress thread has limited use for power transmission.
- It is employed as the thread for light jack screws and vices.

4.3 Torque Required to Raise Load by Square Threaded Screws

- The torque required to raise a load by means of square threaded screw may be determined by considering a screw jack as shown in following fig.
- The load to be raised or lowered is placed on the head of the square threaded rod which is rotated by the application of an effort at the end of lever for lifting or lowering the load.



- A little consideration will show that if one complete turn of a screw thread be imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in following fig.



Let p = Pitch of the screw,

d = Mean diameter of the screw,

α = Helix angle,

P = Effort applied at the circumference of the screw to lift the load,

W = Load to be lifted, and

μ = Coefficient of friction, between the screw and nut

= $\tan \phi$, where ϕ is the friction angle.

From the geometry of the above fig. we find that

$$\tan \alpha = p / \pi d$$

- Since the principle, on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the circumference of a screw jack may be considered to be horizontal as shown in above fig.
- Since the load is being lifted, therefore the force of friction ($F = \mu R_N$) will act downwards. All the forces acting on the body are shown in above Fig.

Resolving the forces along the plane,

$$P \cos \alpha = W \sin \alpha + F = W \sin \alpha + \mu . R_N \qquad ...(i)$$

and resolving the forces perpendicular to the plane,

$$R_N = P \sin \alpha + W \cos \alpha \qquad ...(ii)$$

Substituting this value of R_N in equation (*i*), we have

$$P \cos \alpha = W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha)$$

$$= W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha$$

or $$P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$$

or $$P (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$$

$\therefore$ $$P = W \times \frac{(\sin \alpha + \mu \cos \alpha)}{(\cos \alpha - \mu \sin \alpha)}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

or $$P = W \times \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha}$$

Multiplying the numerator and denominator by $\cos \phi$, we have

$$P = W \times \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \alpha \sin \phi}$$

$$= W \times \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)} = W \tan (\alpha + \phi)$$

∴ Torque required to overcome friction between the screw and nut,

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi)\,\frac{d}{2}$$

When the axial load is taken up by a thrust collar as shown in Fig. 17.2 (*b*), so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$T_2 = \frac{2}{3} \times \mu_1 \times W\left[\frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2}\right]$$

... (Assuming uniform pressure conditions)

$$= \mu_1 \times W\left(\frac{R_1 + R_2}{2}\right) = \mu_1 W R. \quad \text{...(Assuming uniform wear conditions)}$$

where R_1 and R_2 = Outside and inside radii of collar,

R = Mean radius of collar = $\frac{R_1 + R_2}{2}$, and

μ_1 = Coefficient of friction for the collar.

∴ Total torque required to overcome friction (*i.e.*, to rotate the screw),

$$T = T_1 + T_2$$

If an effort P_1 is applied at the end of a lever of arm length l, then the total torque required to overcome friction must be equal to the torque applied at the end of lever, *i.e.*,

$$T = P \times \frac{d}{2} = P_1 \times l$$

4.4 Torque Required to Lower Load by Square Threaded Screws

A little consideration will show that when the load is being lowered, the force of friction ($F = \mu.R_N$) will act upwards. All the forces acting on the body are shown in Fig. 17.4.

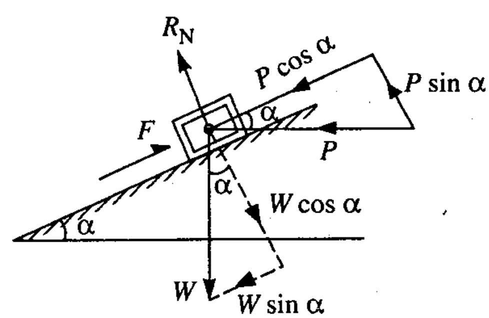


Fig. 17.4

Resolving the forces along the plane,

$$P\cos\alpha = F - W\sin\alpha$$
$$= \mu R_N - W\sin\alpha \quad \text{...(i)}$$

and resolving the forces perpendicular to the plane,

$$R_N = W\cos\alpha - P\sin\alpha \quad \text{...(ii)}$$

Substituting this value of R_N in equation (*i*), we have,

$$P\cos\alpha = \mu(W\cos\alpha - P\sin\alpha) - W\sin\alpha$$
$$= \mu W\cos\alpha - \mu P\sin\alpha - W\sin\alpha$$

or
$$P\cos\alpha + \mu P\sin\alpha = \mu W\cos\alpha - W\sin\alpha$$
$$P(\cos\alpha + \mu\sin\alpha) = W(\mu\cos\alpha - \sin\alpha)$$

or
$$P = W \times \frac{(\mu\cos\alpha - \sin\alpha)}{(\cos\alpha + \mu\sin\alpha)}$$

Substituting the value of $\mu = \tan\phi$ in the above equation, we have

$$P = W \times \frac{(\tan\phi\cos\alpha - \sin\alpha)}{(\cos\alpha + \tan\phi\sin\alpha)}$$

Multiplying the numerator and denominator by cos ϕ, we have

$$P = W \times \frac{(\sin\phi\cos\alpha - \sin\alpha\cos\phi)}{(\cos\alpha\cos\phi + \sin\phi\sin\alpha)}$$

$$= W \times \frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)} = W\tan(\phi - \alpha)$$

∴ Torque required to overcome friction between the screw and nut,

$$T_1 = P \times \frac{d}{2} = W\tan(\phi - \alpha)\frac{d}{2}$$

4.5 Efficiency of Square Threaded Screws

- The efficiency of square threaded screws may be defined as the ratio between the ideal effort (i.e., the effort required to move the load, neglecting friction) to the actual effort (i.e., the effort required to move the load taking friction into account).
- We know that effort applied at the circumference of the screw to lift the load is

$$P = W\tan(\alpha + \phi) \qquad \ldots(i)$$

where W = Load to be lifted,

α = Helix angle,

ϕ = Angle of friction, and

μ = Coefficient of friction between the screw and nut = tan ϕ.

If there would have been no friction between the screw and the nut, then ϕ will be equal to zero. The value of effort P_0 necessary to raise the load, will then be given by the equation,

$$P_0 = W\tan\alpha \qquad [\text{Substituting } \phi = 0 \text{ in equation } (i)]$$

∴ Efficiency,
$$\eta = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{P_0}{P} = \frac{W\tan\alpha}{W\tan(\alpha + \phi)} = \frac{\tan\alpha}{\tan(\alpha + \phi)}$$

This shows that the efficiency of a screw jack, is independent of the load raised.

In the above expression for efficiency, only the screw friction is considered. However, if the screw friction and collar friction is taken into account, then

$$\eta = \frac{\text{Torque required to move the load, neglecting friction}}{\text{Torque required to move the load, including screw and collar friction}}$$

$$= \frac{T_0}{T} = \frac{P_0 \times d/2}{P \times d/2 + \mu_1.W.R}$$

4.6 Maximum Efficiency of a Square Threaded Screw

- We have seen that the efficiency of a square threaded screw is given by –

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)} = \frac{\sin\alpha/\cos\alpha}{\sin(\alpha + \phi)/\cos(\alpha + \phi)} = \frac{\sin\alpha \times \cos(\alpha + \phi)}{\cos\alpha \times \sin(\alpha + \phi)} \qquad \ldots(i)$$

Multiplying the numerator and denominator by 2, we have,

$$\eta = \frac{2\sin\alpha \times \cos(\alpha + \phi)}{2\cos\alpha \times \sin(\alpha + \phi)} = \frac{\sin(2\alpha + \phi) - \sin\phi}{\sin(2\alpha + \phi) + \sin\phi} \qquad \ldots(ii)$$

$$\ldots\left[\because \begin{array}{l} 2\sin A\cos B = \sin(A + B) + \sin(A - B) \\ 2\cos A\sin B = \sin(A + B) - \sin(A - B) \end{array}\right]$$

The efficiency given by equation (*ii*) will be maximum when sin $(2\alpha + \phi)$ is maximum, *i.e.*, when

$$\sin(2\alpha + \phi) = 1 \quad \text{or} \quad \text{when} \quad 2\alpha + \phi = 90°$$

$$\therefore \quad 2\alpha = 90° - \phi \quad \text{or} \quad \alpha = 45° - \phi/2$$

Substituting the value of 2α in equation (*ii*), we have maximum efficiency,

$$\eta_{max} = \frac{\sin(90° - \phi + \phi) - \sin\phi}{\sin(90° - \phi + \phi) + \sin\phi} = \frac{\sin 90° - \sin\phi}{\sin 90° + \sin\phi}$$

$$= \frac{1 - \sin\phi}{1 + \sin\phi}$$

4.7 Example

- A vertical screw with single start square threads of 50 mm mean diameter and 12.5 mm pitch is raised against a load of 10 kN by means of a hand wheel, the boss of which is threaded to act as a nut. The axial load is taken up by a thrust collar which supports the wheel boss and has a mean diameter of 60 mm. The coefficient of friction is 0.15 for the screw and 0.18 for the collar. If the tangential force applied by each hand to the wheel is 100 N, find suitable diameter of the hand wheel.

Solution. Given : $d = 50$ mm ; $p = 12.5$ mm ; $W = 10$ kN $= 10 \times 10^3$ N ; $D = 60$ mm or $R = 30$ mm ; $\mu = \tan\phi = 0.15$; $\mu_1 = 0.18$; $P_1 = 100$ N

We know that $\tan\alpha = \dfrac{p}{\pi d} = \dfrac{12.5}{\pi \times 50} = 0.08$

and the tangential force required at the circumference of the screw,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right]$$

$$= 10 \times 10^3\left[\frac{0.08 + 0.15}{1 - 0.08 \times 0.15}\right] = 2328 \text{ N}$$

We also know that the total torque required to turn the hand wheel,

$$T = P \times \frac{d}{2} + \mu_1 W R = 2328 \times \frac{50}{2} + 0.18 \times 10 \times 10^3 \times 30 \text{ N-mm}$$

$$= 58\,200 + 54\,000 = 112\,200 \text{ N-mm} \qquad \ldots(i)$$

Let D_1 = Diameter of the hand wheel in mm.

We know that the torque applied to the handwheel

$$T = 2P_1 \times \frac{D_1}{2} = 2 \times 100 \times \frac{D_1}{2} = 100\,D_1 \text{ N-mm} \qquad \ldots(ii)$$

Equating equations (*i*) and (*ii*),

$$D_1 = 112\,200 / 100 = 1222 \text{ mm} = 1.222 \text{ m}$$ **Ans.**

4.8 Example

- A vertical two start square threaded screw of a 100 mm mean diameter and 20 mm pitch supports a vertical load of 18 kN. The axial thrust on the screw is taken by a collar bearing of 250 mm outside diameter and 100 mm inside diameter. Find the force required at the end of a lever which is 400 mm long in order to lift and lower the load. The coefficient of friction for the vertical screw and nut is 0.15 and that for collar bearing is 0.20.

Solution. Given : $d = 100$ mm ; $p = 20$ mm ; $W = 18$ kN $= 18 \times 10^3$ N ; $D_2 = 250$ mm or $R_2 = 125$ mm ; $D_1 = 100$ mm or $R_1 = 50$ mm ; $l = 400$ mm ; $\mu = \tan\phi = 0.15$; $\mu_1 = 0.20$

Force required at the end of lever

Let P = Force required at the end of lever.

Since the screw is *A* two start square threaded screw, therefore lead of the screw

$$= 2p = 2 \times 20 = 40 \text{ mm}$$

We know that $\tan\alpha = \dfrac{\text{Lead}}{\pi d} = \dfrac{40}{\pi \times 100} = 0.127$

1. *For raising the load*

We know that tangential force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \tan\phi}\right]$$

$$= 18 \times 10^3 \left[\frac{0.127 + 0.15}{1 - 0.127 \times 0.15}\right] = 5083 \text{ N}$$

and mean radius of the collar,

$$R = \frac{R_1 + R_2}{2} = \frac{50 + 125}{2} = 87.5 \text{ mm}$$

∴ Total torque required at the end of lever,

$$T = P \times \frac{d}{2} + \mu_1 WR$$

$$= 5083 \times \frac{100}{2} + 0.20 \times 18 \times 10^3 \times 87.5 = 569\ 150 \text{ N-mm}$$

We know that torque required at the end of lever (T),

$$569\ 150 = P_1 \times l = P_1 \times 400 \quad \text{or} \quad P_1 = 569\ 150 / 400 = 1423 \text{ N}$$ **Ans.**

2. *For lowering the load*

We know that tangential force required at the circumference of the screw,

$$P = W \tan(\phi - \alpha) = W\left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi \tan\alpha}\right]$$

$$= 18 \times 10^3 \left[\frac{0.15 - 0.127}{1 + 0.15 \times 0.127}\right] = 406.3 \text{ N}$$

and the total torque required the end of lever,

$$T = P \times \frac{d}{2} + \mu_1 WR$$

$$= 406.3 \times \frac{100}{2} + 0.20 \times 18 \times 10^3 \times 87.5 = 335\ 315 \text{ N-mm}$$

We know that torque required at the end of lever (T),

$$335\ 315 = P_1 \times l = P_1 \times 400 \quad \text{or} \quad P_1 = 335\ 315 / 400 = 838.3 \text{ N}$$ **Ans.**

4.9 Over Hauling and Self Locking Screws

- We have seen that the effort required at the circumference of the screw to lower the load is given by –

$$P = W \tan(\phi - \alpha)$$

and the torque required to lower the load,

$$T = P \times \frac{d}{2} = W \tan(\phi - \alpha) \frac{d}{2}$$

- In the above expression, if Ø < α, then torque required to lower the load will be negative.
- In other words, the load will start moving downward without the application of any torque.
- Such a condition is known as over hauling of screws.
- If however, Ø > α, the torque required to lower the load will be positive, indicating that an effort is applied to lower the load.
- Such a screw is known as self locking screw.
- In other words, a screw will be self locking if the friction angle is greater than helix angle or coefficient of friction is greater than tangent of helix angle i.e. tan Ø > tan α .

4.10 Efficiency of Self Locking Screws

We know that the efficiency of screw,

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)}$$

and for self locking screws, $\phi \geq \alpha$ or $\alpha \leq \phi$.

$\therefore$ Efficiency for self locking screws,

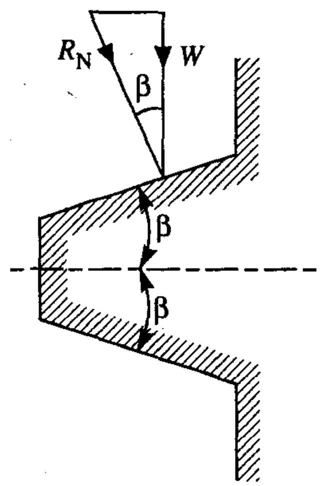
$$\eta \leq \frac{\tan \phi}{\tan(\phi + \phi)} \leq \frac{\tan \phi}{\tan 2\phi} \leq \frac{\tan \phi (1 - \tan^2 \phi)}{2 \tan \phi}$$

$$\leq \frac{1}{2} - \frac{\tan^2 \phi}{2} \qquad \ldots \left(\because \tan 2\phi = \frac{2 \tan \phi}{1 - \tan^2 \phi} \right)$$

- From this expression we see that efficiency of self locking screws is less than 1/2 or 50%. If the efficiency is more than 50%, then the screw is said to be over hauling.

4.11 Acme or Trapezoidal Threads

- We know that the normal reaction in case of a square threaded screw is
  $R_N = W \cos\alpha$, where α is the helix angle.
- But in case of Acme' or trapezoidal thread, the normal reaction between the screw and nut is increased because the axial component of this normal reaction must be equal to the axial load (W).
- Consider an Acme or trapezoidal thread as shown in following Fig.



- Let

  2β= Angle of the Acme thread, and
  β = Semi-angle of the thread.

$$\therefore \qquad R_N = \frac{W}{\cos \beta}$$

and frictional force, $F = \mu . R_N = \mu \times \dfrac{W}{\cos \beta} = \mu_1 . W$

where $\mu / \cos \beta = \mu_1$, known as *virtual coefficient of friction.*

4.12 Example

- The lead screw of a lathe has Acme threads of 50 mm outside diameter and 8 mm pitch. The screw must exert an axial pressure of 2500 N in order to drive the tool carriage. The thrust is carried on a collar 110 mm outside diameter and 55 mm inside diameter and the lead screw rotates at 30 r.p.m. Determine (a) the power required to drive the screw; and (b) the efficiency of the lead screw. Assume a coefficient of friction of 0.15 for the screw and 0.12 for the collar.

Solution. Given : $d_o = 50$ mm ; $p = 8$ mm ; $W = 2500$ N ; $D_1 = 110$ mm or $R_1 = 55$ mm; $D_2 = 55$ mm or $R_2 = 27.5$ mm ; $N = 30$ r.p.m. ; $\mu = \tan\phi = 0.15$; $\mu_2 = 0.12$

(*a*) *Power required to drive the screw*

We know that mean diameter of the screw,

$$d = d_o - p/2 = 50 - 8/2 = 46 \text{ mm}$$

$$\therefore \quad \tan\alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 46} = 0.055$$

Since the angle for Acme threads is $2\beta = 29°$ or $\beta = 14.5°$, therefore virtual coefficient of friction,

$$\mu_1 = \tan\phi_1 = \frac{\mu}{\cos\beta} = \frac{0.15}{\cos 14.5°} = \frac{0.15}{0.9681} = 0.155$$

We know that the force required to overcome friction at the screw,

$$P = W\tan(\alpha + \phi_1) = W\left[\frac{\tan\alpha + \tan\phi_1}{1 - \tan\alpha\,\tan\phi_1}\right]$$

$$= 2500\left[\frac{0.055 + 0.155}{1 - 0.055 \times 0.155}\right] = 530 \text{ N}$$

and torque required to overcome friction at the screw.

$$T_1 = P \times d/2 = 530 \times 46/2 = 12\ 190 \text{ N-mm}$$

We know that mean radius of collar,

$$R = \frac{R_1 + R_2}{2} = \frac{55 + 27.5}{2} = 41.25 \text{ mm}$$

Assuming uniform wear, the torque required to overcome friction at collars,

$$T_2 = \mu_2 W R = 0.12 \times 2500 \times 41.25 = 12\ 375 \text{ N-mm}$$

$\therefore$ Total torque required to overcome friction,

$$= T = T_1 + T_2 = 12\ 190 + 12\ 375 = 24\ 565 \text{ N-mm} = 24.565 \text{ N-m}$$

We know that power required to drive the screw

$$= T.\omega = \frac{T \times 2\pi N}{60} = \frac{24.565 \times 2\pi \times 30}{60} = 77\text{W} = 0.077 \text{ kW} \quad \textbf{Ans.}$$

... ($\because \omega = 2\pi N/60$)

(*b*) *Efficiency of the lead screw*

We know that the torque required to drive the screw with no friction,

$$T_o = W\tan\alpha \times \frac{d}{2} = 2500 \times 0.055 \times \frac{46}{2} = 3163 \text{ N-mm}$$

$$= 3.163 \text{ N-m}$$

$\therefore$ Efficiency of the lead screw,

$$\eta = \frac{T_o}{T} = \frac{3.163}{24.565} = 0.13 \text{ or } 13\% \quad \textbf{Ans.}$$

4.13 Stresses in Power Screws

Following types of stresses are induced in the screw.

a. Direct tensile or compressive stress due to an axial load.

- The direct stress due to the axial load may be determined by dividing the axial load (W) by the minimum cross-sectional area of the screw (A_c) i.e., area corresponding to minor or core diameter (d_c).
- Direct stress (tensile or compressive) is given as –

$$\sigma t = \frac{W}{Ac}$$

- This is only applicable when the axial load is compressive and the unsupported length of the screw between the load and the nut is short.
- But when the screw is axially loaded in compression and the unsupported length of the screw between the load and the nut is too great, then the design must be based on column theory assuming suitable end conditions.
- In such cases the cross-sectional area corresponding to core diameter may be obtained by using Rankine-Gordn formula or J.B. Johnson's formula.
- According to this,

$$W_{cr} = A_c \times \sigma_y \left[1 - \frac{\sigma_y}{4 C \pi^2 E}\left(\frac{L}{k}\right)^2\right]$$

$$\therefore \quad \sigma_c = \frac{W}{A_c}\left[\frac{1}{1 - \frac{\sigma_y}{4 C \pi^2 E}\left(\frac{L}{k}\right)^2}\right]$$

where W_{cr} = Critical load,

σ_y = Yield stress,

L = Length of screw,

k = Least radius of gyration,

C = End-fixity coefficient,

E = Modulus of elasticity, and

σ_c = Stress induced due to load W.

b. Torsional shear stress.

- Since the screw is subjected to a twisting moment, therefore torsional shear stress is induced. This is obtained by considering the minimum cross-section of the screw.
- We know that torque transmitted by the screw-

$$T = \frac{\pi}{16} \times \tau (d_c)^3$$

or shear stress induced,

$$\tau = \frac{16 T}{\pi (d_c)^3}$$

- When the screw is subjected to both direct stress and torsional shear stress, then the design must be based on maximum shear stress theory.
- According to which maximum shear stress on the minor diameter section is given by

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_t \text{ or } \sigma_c)^2 + 4\tau^2}$$

c. Shear stress due to axial load.

- The threads of the screw at the core or root diameter and the threads of the nut at the major diameter may shear due to the axial load.
- Assuming that the load is uniformly distributed over the threads in contact, we have shear stress for screw,

$$\tau_{(screw)} = \frac{W}{\pi n \cdot d \cdot t}$$

- and shear stress for nut,

$$\tau_{(nut)} = \frac{W}{\pi n \cdot d_o \cdot t}$$

Where ,

W = Axial load on the screw,
n = Number of threads in engagement,
d_c = Core or root diameter of the screw,
d_o = Outside or major diameter of nut or screw, and
t = Thickness or width of thread.

d. Bearing pressure.

- In order to reduce wear of the screw and nut, the bearing pressure on the thread surfaces must be within limits.
- In the design of power screws, the bearing pressure depends upon the materials of the screw and nut, relative velocity between the nut and screw and the nature of lubrication.
- Assuming that the load is uniformly distributed over the threads in contact, the bearing pressure on the threads is given by

$$p_b = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n} = \frac{*W}{\pi d \cdot t \cdot n}$$

Where ,

d = Mean diameter of screw,
t = Thickness or width of screw = $p/2$, and
n = Number of threads in contact with the nut $= \frac{\text{Height of the nut}}{\text{Pitch of threads}} = \frac{h}{p}$

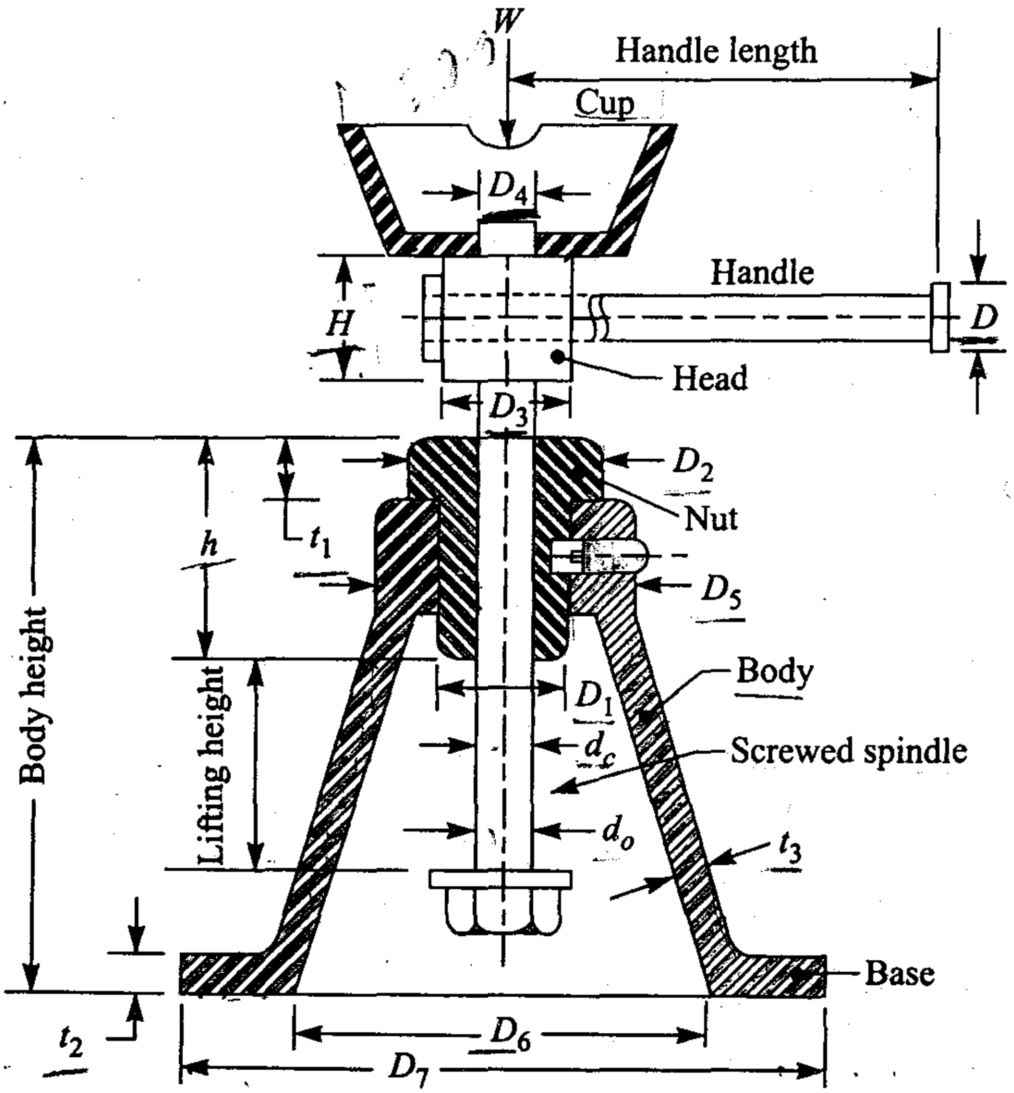
4.14 Design of Screw Jack

- A bottle screw jack for lifting loads is shown in following fig.
- The various parts of the screw jack are as follows:
  1. Screwed spindle having square threaded screws,
  2. Nut and collar for nut,
  3. Head at the top of the screwed spindle for handle,
  4. Cup at the top of head for the load, and
  5. Body of the screw jack.

Step 1

- First of all, find the core diameter(d_c) by considering that the screw is under pure compression, i.e.,

$$\sigma c = \frac{P}{\frac{\pi dc^2}{4}}$$



Step 2

Find the torque (T_1) required to rotate the screw and find the shear stress (τ) due to this torque.

We know that the torque required to lift the load,

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi) \frac{d}{2}$$

where P = Effort required at the circumference of the screw, and
d = Mean diameter of the screw.

∴ Shear stress due to torque T_1,

$$\tau = \frac{16\, T_1}{\pi (d_c)^3}$$

Also find direct compressive stress (σ_c) due to axial load, *i.e.*,

$$\sigma_c = \frac{W}{\frac{\pi}{4}(d_c)^2}$$

Step 3

- Find the principal stresses as follows:
- Maximum principal stress (tensile or compressive),

$$\sigma_{c(max)} = \frac{1}{2}\left[\sigma_c + \sqrt{(\sigma_c)^2 + 4\tau^2}\right]$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2}$$

- These stresses should be less than the permissible stresses.

Step 4

- Find the height of nut (h), considering the bearing pressure on the nut. We know that the bearing pressure on the nut,

$$p_b = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right]n}$$

Where, n = Number of threads in contact with screwed spindle.

Height of nut, h = n xp
Where , p = Pitch of threads.

Step 5

- Check the stressess in the screw and nut as follows

$$\tau_{(screw)} = \frac{W}{\pi\, n.d_c.t}$$

$$\tau_{(nut)} = \frac{W}{\pi\, n.d_o.t}$$

where , t = thickness of screw = p / 2

Step 6

- Find inner diameter (D_1), outer diameter (D_2) and thickness (t_1) of the nut collar.
- The inner diameter (D_1) is found by considering the tearing strength of the nut.
  We know that

$$\sigma t = \frac{W}{\frac{\pi}{4}\left[(D_1)^2 - (d_o)^2\right]}$$

- The, outer diameter (D_2) is found by considering the crushing strength of the nut collar.
  We know that - -

$$\sigma c = \frac{W}{\frac{\pi}{4}\left[(D_2)^2 - (D_1)^2\right]}$$

- The thickness (t_1) of the nut collar is foud by considering the shearing strength of the nut collar.
  We know that--

$$\tau = \frac{W}{\pi.D1.t1}$$

Step 7

- Fix the dimensions for the-diameter of head (D_3) on the top of the screw and for the cup.
- Take $\mathbf{D_3 = 1.75\ d_0}$.
- The seat for the cup is made equal to the diameter of head and it is chamfered at the top.
- The cup is fitted with a pin of diameter $\mathbf{D_4 = D_3 /4}$ approximately. This pin remains a loose fit in the cup.

Step 8

- Find the torque required (T_2) to overcome friction at the top of screw. We know that –

$$T_2 = \frac{2}{3} \times \mu_1 W \left[\frac{(R_3)^3 - (R_4)^3}{(R_3)^2 - (R_4)^2}\right] \text{ ... (Assuming uniform pressure conditions)}$$

$$= \mu_1 W \left(\frac{R_3 + R_4}{2}\right) = \mu_1 W R \text{ ... (Assuming uniform wear conditions)}$$

Where ,

R_3 = Radius of head
R_4 = Radius of pin

Step 9

- Now the total torque to which the handle will be subjected is given by
  $T = T_1 + T_2$
- Assuming that a person can apply a force of 300 — 400 N intermittently, the length of handle required is -
  l = T/300
  The length of handle may be fixed by giving some allowance for gripping.

Step 10

- The diameter of handle (D) may be obtained by considering bending effects. We know that bending moment,

$$M = \frac{\pi}{32} \times \sigma_b \times D^3$$

Step 11

- The height of head (H) is usually taken as twice the diameter of handle, i.e., H = 2D.

Step 12

Now check the screw for buckling load.

Effective length or unsupported length of the screw,

$$L = \text{Lift of screw} + \tfrac{1}{2}\text{ Height of nut}$$

We know that buckling or critical load,

$$W_{cr} = A_c.\sigma_y \left[1 - \frac{\sigma_y}{4\,C\pi^2 E}\left(\frac{L}{k}\right)^2\right]$$

where σ_y = Yield stress,

C = End fixity coefficient. The screw is considered to be a strut with lower end fixed and load end free. For one end fixed and the other end free, $C = 0.25$

k = Radius of gyration = $0.25\, d_c$

Step 13

- Fix the dimensions for the body of the screw jack.
- Find efficiency of the screw jack.

Example

A screw jack is to lift a load of 80 kN through a height of 400 mm. The elastic strength of screw material in tension and compression is 200 MPa and in shear 10 MPa. The material for nut is phosphor-bronze for which the elastic limit may be taken as 100 MPa in tension, 90 MPa in compression and 80 MPa in shear .The bearing pressure between the nut and the screw is not to exceed 18 N/mm2. Design and draw the screw jack. The design should include the design of 1. screw, 2. nut, 3. handle and cup,and 4. body.

Solution. Given : $W = 80$ kN $= 80 \times 10^3$N ; $H_1 = 400$ mm $= 0.4$ m ; $\sigma_{et} = \sigma_{ec} = 200$ MPa $= 200$ N/mm^2; $\tau_e = 120$ MPa $= 120$ N/mm^2 ; $\sigma_{et(nut)} = 100$ MPa $= 100$ N/mm^2 ; $\sigma_{ec\ (nut)} = 90$ MPa $= 90$ N/mm^2 ; $\tau_{e(nut)} = 80$ MPa $= 80$N/mm^2 ; $p_b = 18$ N/mm^2

The various parts of a screw jack are designed as discussed below :

1. *Design of screw for spindle*

Let d_c = Core diameter of the screw.

Since the screw is under compression, therefore load (W),

$$80 \times 10^3 = \frac{\pi}{4}(d_c)^2 \times \frac{\sigma_{ec}}{F.S.} = \frac{\pi}{4}(d_c)^2 \frac{200}{2} = 78.55\ (d_c)^2$$

... (Taking factor of safety, $F.S. = 2$)

$\therefore$ $(d_c)^2 = 80 \times 10^3 / 78.55 = 1018.5$ or $d_c = 32$ mm

But for safety purpose adding 6 mm to the dimensions ,

$d_c = 38$ mm

$d_o = 46$ mm and

pitch = 8 mm

Now let us check for principal stresses :

We know that the mean diameter of screw,

$$d = \frac{d_o + d_c}{2} = \frac{46 + 38}{2} = 42 \text{ mm}$$

and $$\tan\alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 42} = 0.0606$$

Assuming coefficient of friction between screw and nut,

$\mu = \tan\varnothing = 0.14$

$\therefore$ Torque required to rotate the screw in the nut,

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi)\frac{d}{2} = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\ \tan\phi}\right]\frac{d}{2}$$

$$= 80 \times 10^3 \left[\frac{0.0606 + 0.14}{1 - 0.0606 \times 0.14}\right]\frac{42}{2} = 340 \times 10^3 \text{ N-mm}$$

Now compressive stress due to axial load,

$$\sigma_c = \frac{W}{A_c} = \frac{W}{\frac{\pi}{4}(d_c)^2} = \frac{80 \times 10^3}{\frac{\pi}{4}(38)^2} = 70.53 \text{ N/mm}^2$$

and shear stress due to the torque,

$$\tau = \frac{16\,T_1}{\pi (d_c)^3} = \frac{16 \times 340 \times 10^3}{\pi (38)^3} = 31.55 \text{ N/mm}^2$$

$\therefore$ Maximum principal stress (tensile or compressive),

$$\sigma_{c(max)} = \frac{1}{2}\left[\sigma_c + \sqrt{(\sigma_c)^2 + 4\tau^2}\right] = \frac{1}{2}\left[70.53 + \sqrt{(70.53)^2 + 4\,(31.55)^2}\right]$$

$$= \frac{1}{2}\left[70.53 + 94.63\right] = 82.58 \text{ N/mm}^2$$

The given value of σ_c is equal to $\frac{\sigma_{ec}}{F.S.}$, *i.e.*, $\frac{200}{2} = 100$ N/mm^2.

We know that maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(70.53)^2 + 4\,(31.55)^2}\right]$$

$$= \frac{1}{2} \times 94.63 = 47.315 \text{ N/mm}^2$$

The given value of τ is equal to $\frac{\tau_e}{F.S.}$, *i.e.*, $\frac{120}{2} = 60$ N/mm².

Since these maximum stresses are within limits, therefore design of screw for spindle is safe.

2. *Design for nut*

Let n = Number of threads in contact with the screwed spindle,

h = Height of nut = $n \times p$, and

t = Thickness of screw = $p/2 = 8/2 = 4$ mm

Assume that the load is distributed uniformly over the cross-sectional area of nut.

We know that the bearing pressure (p_b),

$$18 = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right]n} = \frac{80 \times 10^3}{\frac{\pi}{4}\left[(46)^2 - (38)^2\right]n} = \frac{151.6}{n}$$

$\therefore$ $n = 151.6 / 18 = 8.4$ say 10 threads **Ans.**

and height of nut, $h = n \times p = 10 \times 8 = 80$ mm **Ans.**

Now, let us check the stresses induced in the screw and nut.

We know that shear stress in the screw,

$$\tau_{(screw)} = \frac{W}{\pi\, n.d_c.t} = \frac{80 \times 10^3}{\pi \times 10 \times 38 \times 4} = 16.75 \text{ N/mm}^2$$

... ($\because$ $t = p/2 = 4$ mm)

and shear stress in the nut,

$$\tau_{(nut)} = \frac{W}{\pi\, n.d_o\, t} = \frac{80 \times 10^3}{\pi \times 10 \times 46 \times 4} = 13.84 \text{ N/mm}^2$$

Since these stresses are within permissible limit, therefore design for nut is safe.

Let D_1 = Outer diameter of nut,

D_2 = Outside diameter for nut collar, and

t_1 = Thickness of nut collar.

First of all considering the tearing strength of nut, we have

$$W = \frac{\pi}{4}\left[(D_1)^2 - (d_o)^2\right]\sigma_t$$

$$80 \times 10^3 = \frac{\pi}{4}\left[(D_1)^2 - (46)^2\right]\frac{100}{2} = 39.3\left[(D_1)^2 - 2116\right]$$

or $(D_1)^2 - 2116 = 80 \times 10^3 / 39.3 = 2036$

$\therefore$ $(D_1)^2 = 2036 + 2116 = 4152$ or $D_1 = 65$ mm **Ans.**

Now considering the crushing of the collar of the nut, we have

$$W = \frac{\pi}{4}\left[(D_2)^2 - (D_1)^2\right]\sigma_c$$

$$80 \times 10^3 = \frac{\pi}{4}\left[(D_2)^2 - (65)^2\right]\frac{90}{2} = 35.3\left[(D_2)^2 - 4225\right] \quad \ldots \left[\because \sigma_c = \frac{\sigma_{ec\,(nut)}}{F.S.}\right]$$

or $(D_2)^2 - 4225 = 80 \times 10^3 / 35.3 = 2266$

$\therefore$ $(D_2)^2 = 2266 + 4225 = 6491$ or $D_2 = 80.6$ say 82 mm **Ans.**

Considering the shearing of the collar of the nut, we have

$$W = \pi D_1 \times t_1 \times \tau$$

$$80 \times 10^3 = \pi \times 65 \times t_1 \times \frac{80}{2} = 8170\, t_1 \qquad \ldots \left[\because \tau = \frac{\tau_{e\,(nut)}}{F.S.}\right]$$

$\therefore$ $$t_1 = 80 \times 10^3 / 8170 = 9.8 \text{ say } 10 \text{ mm} \quad \textbf{Ans.}$$

3. *Design for handle and cup*

The diameter of the head (D_3) on the top of the screwed rod is usually taken as 1.75 times the outside diameter of the screw (d_o).

$\therefore$ $$D_3 = 1.75\, d_o = 1.75 \times 46 = 80.5 \text{ say } 82 \text{ mm} \quad \textbf{Ans.}$$

The head is provided with two holes at the right angles to receive the handle for rotating the screw. The seat for the cup is made equal to the diameter of head, *i.e.*, 82 mm and it is given chamfer at the top. The cup prevents the load from rotating. The cup is fitted to the head with a pin of diameter D_4 = 20 mm. The pin remains loose fit in the cup. Other dimensions for the cup may be taken as follows :

Height of cup = 50 mm **Ans.**

Thickness of cup = 10 mm **Ans.**

Diameter at the top of cup = 160 mm **Ans.**